**ME 55600/I0200**

**Homework #2: Exact Solutions**

1. Determine the velocity profile for Couette flow between two horizontal flat plates separated by a distance H, when the top plate is moving at a velocity V and the bottom plate is at rest. Consider two conditions: (i) zero pressure gradient, and (ii) non-zero pressure gradient. Draw the velocity profile for the following three cases:

= 0, 0, 0

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Couette Flow

![Diagram, engineering drawing

Description automatically generated]()

The governing equations are:

(1)

(2)

Boundary Conditions:

1. at
2. at

Integrate (2) : ; then

Substitute into (1):

Since each side is a function of a different independent variable, both sides must be equal to a constant,

Therefore: ;

Apply the pressure boundary conditions:

From (i):

From (ii):

Therefore, the solution for the pressure is

Apply the velocity boundary conditions:

From (iii):

From (iv): ; Then:

The solution for the velocity is:

Special cases:

1. No pressure gradient

|  |  |
| --- | --- |
|  |  |

1. Adverse pressure gradient

|  |  |
| --- | --- |
|  | Diagram  Description automatically generated |

1. Negative pressure gradient

|  |  |
| --- | --- |
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1. Two concentric cylinders of radii and re rotating with angular velocities and , respectively. Solve the Navier-Stokes equations for the velocity profile .

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Flow between rotating cylinders

![Diagram, engineering drawing

Description automatically generated]()

Governing Equations

(1)

(2)

The velocity can be obtained from (1). The pressure can be determined from (2) once the velocity is known.

B.C.

(i)



Solution of (1): or

Then:

Apply B.C. and solve for the two constant coefficients

Substitute the constant coefficients into the solution to get

or